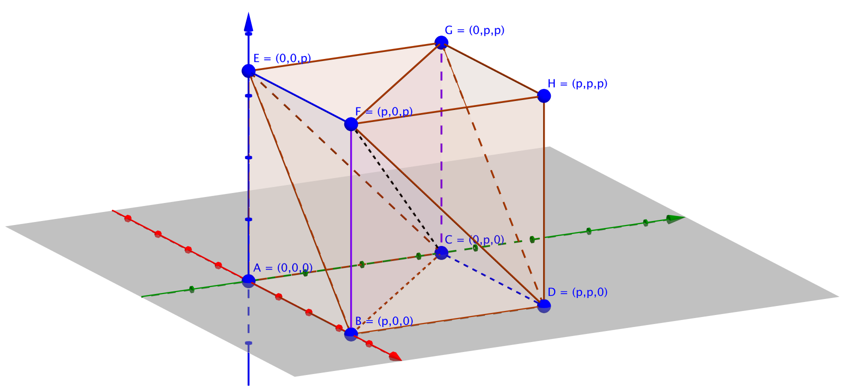
ANAGLYPH PICTURES IN MY RESEARCH **Whitney Berard**

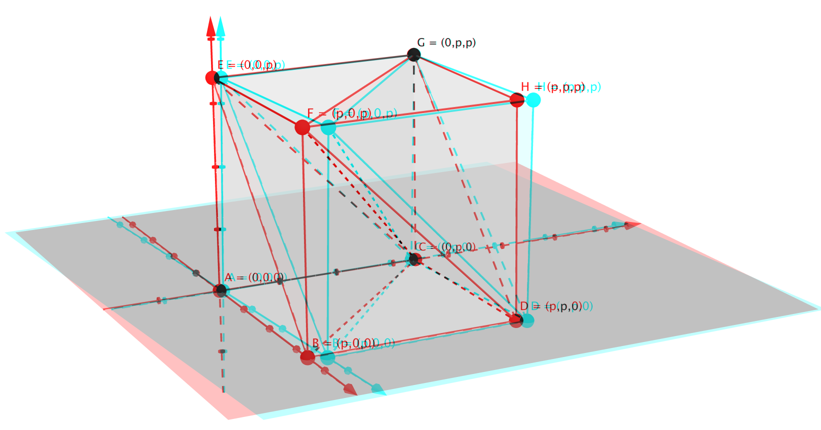
This is a brief description of one of the visual aspects of my recent research. For a more detailed description of my research, and for an interactive version of the final graph below, please visit my website at `www.whitneyberard.com/research`.

The objects I study are called **Serre weights** — irreducible $\mathbb{F}_p$-representations of $\mathrm{GL}_n(\mathbb{F}_p)$. By a theorem of Steinberg, these can be parametrized by n-tuples of integers $(a_1, \ldots, a_n)$, where $0 \leq a_i - a_{i+1} \leq p - 1$, and we write $F(a_1, \ldots, a_n)$ to represent a Serre weight.

For four-dimensional representations, we visualize a weight $F(a-3, b-2, c-1, d)$ as the point $(a-b, b-c, c-d)$ in 3D space. The **weight space** we are concerned with can be visualized as a cube with side length p, with four planes cutting the cube into six pieces, called alcoves. The only way I have found to effectively understand this picture is with a 3D picture viewable by 3D glasses.

Below is a picture of the dimension four alcoves, followed by a similar picture rendered to be seen with 3D red-cyan glasses.



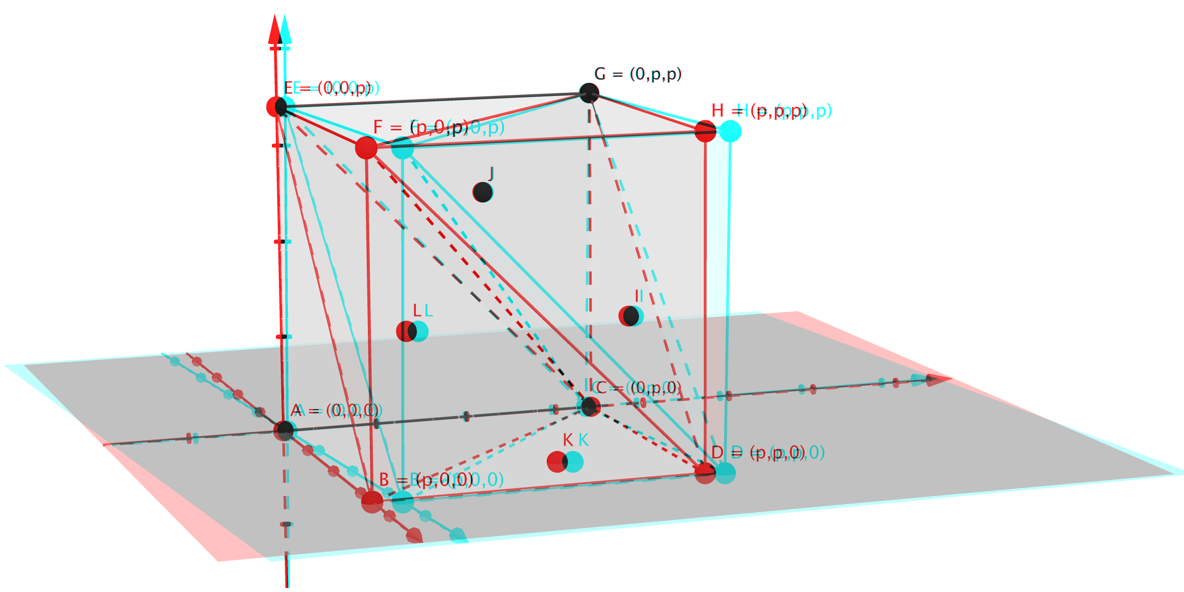


For a particular n-tuple $\mathbf{a} = (a_1, \ldots, a_n)$, I am interested in the list of Jordan-Hölder factors $F(\mathbf{a}_i)$ of a Weyl module $W(\mathbf{a})$. It turns out that each of the $\mathbf{a}_i$'s is a successive reflection of $\mathbf{a}$ about the planes in the relevant weight space.

For me, the interesting cases to look at are when a weight lies on one of the boundaries. For example, let's look at the weight $\mathbf{a} = (a-3, b-2, c-1, d)$ with $(a-b, b-c, c-d) = (x, p, x)$, where $x < p/2$. To find the Jordan-Hölder factors of $W(a-3, b-2, c-1, d)$, we look at all the reflections of $\mathbf{a}$ about the hyperplanes inside this cube. When $\mathbf{a}$ is on a boundary, it happens that the only Jordan-Hölder factors that survive are those that end up on an "upper" boundary of an alcove.

In the following picture, we take the point $I = (x, p, x)$, and reflect it about the various hyperplanes to obtain J, K, L. If you've got your trusty 3D glasses on, you'll notice that:

- $I = (x, p, x)$ is on an upper boundary of the alcove $FGCD$,
- $J = (0, p - x, 2x)$ is on a lower boundary of the alcove $EFGC$,
- $K = (2x, p - a, 0)$ is on a lower boundary of the alcove $BCDF$, and
- $L = (a, p - 2a, a)$ is on the upper boundary of the alcove $ABCE$.



Since J, K are not on upper boundaries of any alcove, those Serre weights do not survive in the Jordan-Hölder decomposition. The Jordan-Holder factors of $W(\mathbf{a})$ are $F(\mathbf{a})$ and $F(\mathbf{b})$, where $\mathbf{b}$ is the Serre weight corresponding to the point L in the picture.